Status of the YN interaction in chiral EFT

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YN in chiral effective field theory

We follow the scheme of S. Weinberg (1990)
in complete analogy to the study of NN in χEFT by E. Epelbaum et al.

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three baryon forces and external current operators in a consistent way

Obstacle: YN data base is rather poor

- about 35 data points, all from the 1960s
- 10 data points from the KEK-PS E251 collaboration (1999-2005)
  (cf. $>$ 4000 NN data for $E_{lab} < 350$ MeV!)
- constraints from hypernuclei
- no polarization data $\Rightarrow$ no phase shift analysis

$\rightarrow$ impose $SU(3)_f$ constraints

Power counting

$$V_{\rm eff} \equiv V_{\rm eff}(Q, g, \mu) = \sum_{\nu} (Q/\Lambda)^{\nu} \, \mathcal{V}_{\nu}(Q/\mu, g)$$

- Q ... soft scale (baryon three-momentum, Goldstone boson four-momentum, Goldstone boson mass)
- Λ ... hard scale
- g ... pertinent low–energy constants
- μ ... regularization scale
- $\mathcal{V}_{\nu}$ ... function of order one
- $\nu \geq 0$ ... chiral power

Leading order (LO): $\nu = 0$

a) non-derivative four-baryon contact terms

b) one-meson (Goldstone boson) exchange diagrams

Next-to-leading order (NLO): $\nu = 2$

a) four-baryon contact terms with two derivatives

b) two-meson (Goldstone boson) exchange diagrams

Contact terms for BB

e.g., LO contact terms for BB:

$$\mathcal{L} = C_i \left(\bar{N}\Gamma_i N\right)\left(\bar{N}\Gamma_i N\right) \quad\Rightarrow\quad \begin{aligned} \mathcal{L}^1 &= \tilde{C}_i^1 \left\langle \bar{B}_a \bar{B}_b \left(\Gamma_i B\right)_b \left(\Gamma_i B\right)_a \right\rangle , \\ \mathcal{L}^2 &= \tilde{C}_i^2 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle , \\ \mathcal{L}^3 &= \tilde{C}_i^3 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \right\rangle \left\langle \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle \end{aligned}$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

a, b ... Dirac indices of the particles

$$\Gamma_1 = 1\,,\ \Gamma_2 = \gamma^\mu\,,\ \Gamma_3 = \sigma^{\mu\nu}\,,\ \Gamma_4 = \gamma^\mu\gamma_5\,,\ \Gamma_5 = \gamma_5$$

C_i, $\tilde{C}_i$ ... low-energy coefficients

Contact terms for BB

spin-momentum structure of the contact term potential:

BB contact terms without derivatives (LO):

$$V^{(0)}_{BB\to BB} = C_{S,BB\to BB} + C_{T,BB\to BB}\,\vec\sigma_1\cdot\vec\sigma_2$$

BB contact terms with two derivatives (NLO):

$$\begin{aligned} V^{(2)}_{BB\to BB} &= C_1\vec q^{\,2} + C_2\vec k^{\,2} + (C_3\vec q^{\,2} + C_4\vec k^{\,2})(\vec\sigma_1\cdot\vec\sigma_2) \\ &+ \frac{i}{2}C_5(\vec\sigma_1+\vec\sigma_2)\cdot(\vec q\times\vec k) + C_6(\vec q\cdot\vec\sigma_1)(\vec q\cdot\vec\sigma_2) \\ &+ C_7(\vec k\cdot\vec\sigma_1)(\vec k\cdot\vec\sigma_2) + \frac{i}{2}C_8(\vec\sigma_1-\vec\sigma_2)\cdot(\vec q\times\vec k) \end{aligned}$$

note: $C_i \longrightarrow C_{i,BB\to BB}$

$\vec q = \vec p\,' - \vec p$; $\vec k = (\vec p\,' + \vec p)/2$

$SU(3)$ symmetry

10 independent spin-isospin channels in NN and YN (for L=0)
(NN (I=0), NN (I=1), ΛN, ΣN (I=1/2), ΣN (I=3/2), $\Lambda N \leftrightarrow \Sigma N$)

$\Rightarrow$ in principle (at LO), 10 low-energy constants

$SU(3)$ symmetry $\Rightarrow$ only 5 independent low-energy constants

$SU(3)$ structure for scattering of two octet baryons:
direct product:

$$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$$

$C_{S,i}$, $C_{T,i}$, $C_{1,i}$, etc., can be expressed by the coefficients corresponding to the $SU(3)_f$ irreducible representations:
C^1, C^{8_a}, C^{8_s}, C^{10^*}, C^{10}, C^{27}

$SU(3)$ structure of contact terms for BB

	Channel	I	V_{1S0}, $V_{3P0,3P1,3P2}$	V_{3S1}, $V_{3S1-3D1}$, V_{1P1}	$V_{1P1-3P1}$
$S = 0$	$NN \to NN$	0	–	C^{10^*}	–
	$NN \to NN$	1	C^{27}	–	–
$S = -1$	$\Lambda N \to \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{-1}{\sqrt{20}} C^{8_s8_a}$
	$\Lambda N \to \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(-C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}} C^{8_s8_a}$
					$\frac{-1}{\sqrt{20}} C^{8_s8_a}$
	$\Sigma N \to \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27} + 9C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}} C^{8_s8_a}$
	$\Sigma N \to \Sigma N$	$\frac{3}{2}$	C^{27}	C^{10}	–

Number of contact terms:

NN: 2 (LO) 7 (NLO)
YN: +3 (LO) +11 (NLO)
YY: +1 (LO) + 4 (NLO)

$\Rightarrow C^1$ contributes only to $I = 0$, $S = -2$ channels!!

Pseudoscalar-meson exchange

$\mathrm{SU}(3)_f$-invariant pseudoscalar-meson-baryon interaction Lagrangian:

$$\mathcal{L} = -\left\langle \frac{g_A(1-\alpha)}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5\left\{\partial_\mu P, B\right\} + \frac{g_A\alpha}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5\left[\partial_\mu P, B\right]\right\rangle$$

$$f = g_A/(2F_\pi);\quad g_A \simeq 1.26,\quad F_\pi \approx 93 \text{ MeV}$$

$$\alpha = F/(F+D) \text{ with } g_A = F + D$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

$$\begin{aligned}
f_{NN\pi} &= f & f_{NN\eta_8} &= \frac{1}{\sqrt{3}}(4\alpha-1)f & f_{\Lambda NK} &= -\frac{1}{\sqrt{3}}(1+2\alpha)f \\
f_{\Xi\Xi\pi} &= -(1-2\alpha)f & f_{\Xi\Xi\eta_8} &= -\frac{1}{\sqrt{3}}(1+2\alpha)f & f_{\Xi\Lambda K} &= \frac{1}{\sqrt{3}}(4\alpha-1)f \\
f_{\Lambda\Sigma\pi} &= \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma\Sigma\eta_8} &= \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma NK} &= (1-2\alpha)f \\
f_{\Sigma\Sigma\pi} &= 2\alpha f & f_{\Lambda\Lambda\eta_8} &= -\frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Xi\Sigma K} &= -f
\end{aligned}$$

Pseudoscalar-meson (boson) exchange

One-pseudoscalar-meson exchange (V^{OBE}) [LO]

$$V^{OBE}_{B_1 B_2 \to B_1' B_2'} = -f_{B_1 B_1' P} f_{B_2 B_2' P} \frac{\left(\vec{\sigma}_1 \cdot \vec{q}\right)\left(\vec{\sigma}_2 \cdot \vec{q}\right)}{\vec{q}^{\,2} + m_P^2}$$

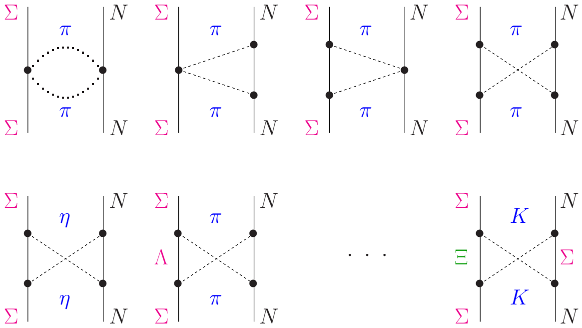
$f_{B_1 B_1' P}$ ... coupling constants
m_P ... mass of the exchanged pseudoscalar meson

- dynamical breaking of $SU(3)$ symmetry due to the mass splitting of the ps mesons
  (m_π = 138.0 MeV, m_K = 495.7 MeV, m_η = 547.3 MeV)
  taken into account already at LO!

Details:

(H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244; PLB 653 (2007) 29)

Two-pseudoscalar-meson exchange diagrams (V^{TBE}) [NLO]



⇒ J.H., S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, NPA 915 (2013) 24

Coupled channels Lippmann-Schwinger Equation

$$T_{\rho'\rho}{}^{\nu'\nu,J}(p',p) = V_{\rho'\rho}{}^{\nu'\nu,J}(p',p) + \sum_{\rho'',\nu''}\int_0^\infty \frac{dp''p''^2}{(2\pi)^3}\, V_{\rho'\rho''}{}^{\nu'\nu'',J}(p',p'')\frac{2\mu_{\nu''}}{p^2-p''^2+i\eta}\,T_{\rho''\rho}{}^{\nu''\nu,J}(p'',p)$$

ρ', ρ = ΛN, ΣN

LS equation is solved for particle channels (in momentum space)

Coulomb interaction is included via the Vincent-Phatak method

The potential in the LS equation is cut off with the regulator function:

$$V_{\rho'\rho}{}^{\nu'\nu,J}(p',p) \to f^\Lambda(p')V_{\rho'\rho}{}^{\nu'\nu,J}(p',p)f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^4}$$

consider values Λ = 450 - 700 MeV [500 - 650 MeV]

NLO Results

Pseudoscalar-meson exchange

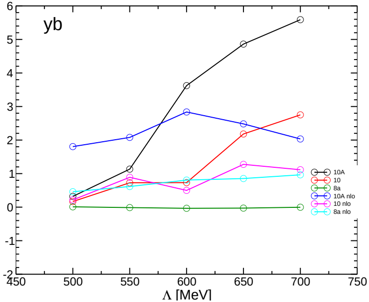
- All one- and two-pseudoscalar-meson exchange diagrams are included
- $SU(3)$ symmetry is broken by using the physical masses of the pion, kaon, and eta
- $SU(3)$ breaking in the coupling constants is ignored
  $F_\pi = F_K = F_\eta = F_0 = 93$ MeV; $g_A = 1.26$
- assume that $\eta \equiv \eta_8$ (i.e. $\theta_P = 0^o$ and $f_{BB\eta_1} \equiv 0$)
- assume that $\alpha = F/(F+D) = 2/5$
  (semi-leptonic decays $\Rightarrow \alpha \approx 0.364$)
- Correction to V^{OBE} due to baryon mass differences are ignored
- (A fit with two-pion-meson exchange diagrams is possible!)
- (A fit with physical values for F_π, F_K, F_η is possible!)

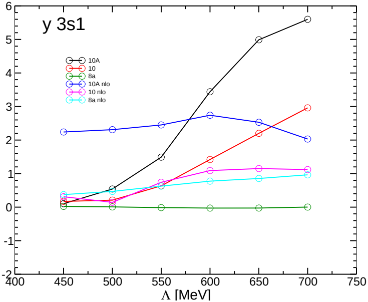
Contact terms

- $SU(3)$ symmetry is assumed
- (at NLO $SU(3)$ breaking corrections to the LO contact terms arise!)
- 10 contact terms in S-waves
  no SU(3) constraints from the NN sector are imposed!
- 12 contact terms in P-waves and in ${}^3S_1 - {}^3D_1$
  SU(3) constraints from the NN sector are imposed!
- 1 contact term in ${}^1P_1 - {}^3P_1$ (singlet-triplet mixing) is set to zero

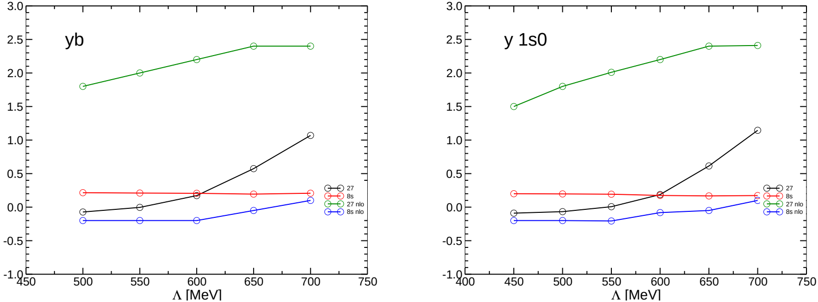
- contact terms in S-waves: can be fairly well fixed from data
- some correlations between NLO and LO LECs

Contact terms (old - new)





Contact terms (old - new)



Contact terms in P-waves

- contact terms in P-waves are much less constrained
- use SU(3) and fix some (5) LECs from NN
- the others (7) are fixed from "bulk" properties:

(1) $\sigma_{\Lambda p} \approx 10$ mb at $p_{lab} \approx 700 - 900$ MeV/c

(2) $d\sigma/d\Omega_{\Sigma^- p \to \Lambda n}$ at $p_{lab} \approx 135 - 160$ MeV/c

Other (future) options:

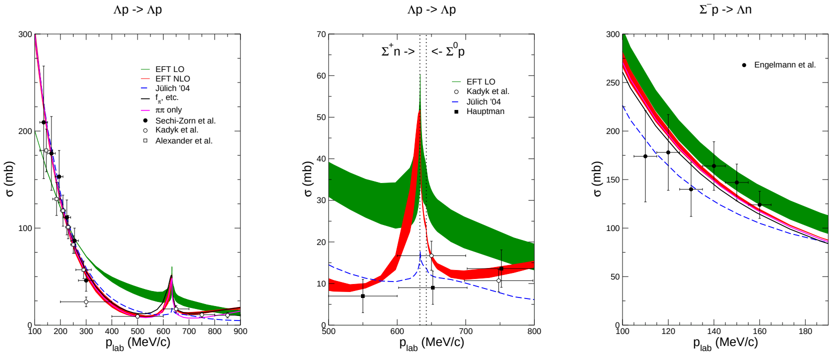
- consider matter properties:

use spin-orbit splitting of the Λ single particle levels in nuclei

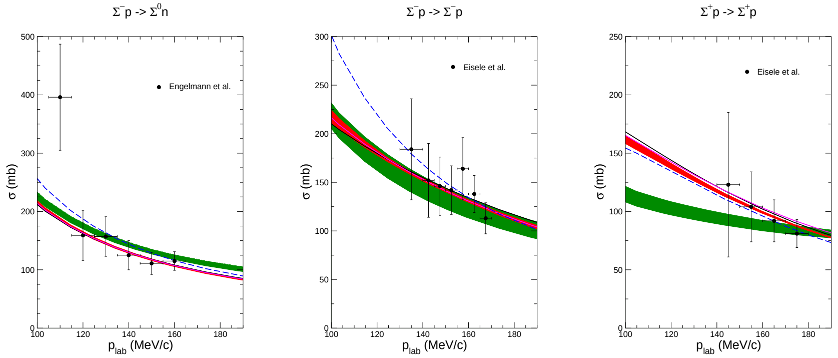
Consider the Scheerbaum factor S_Λ calculated in nuclear matter to relate the strength of the Λ-nucleus spin-orbit potential to the two body ΛN interaction

(R.R. Scheerbaum, NPA 257 (1976) 77)

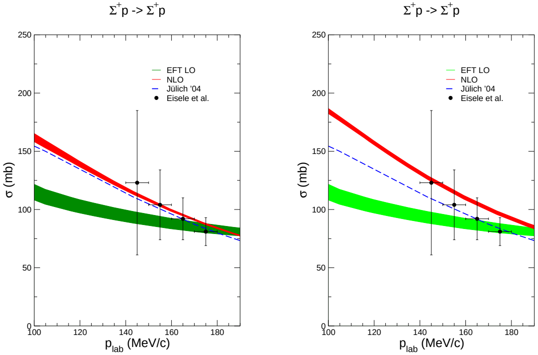
YN integrated cross sections



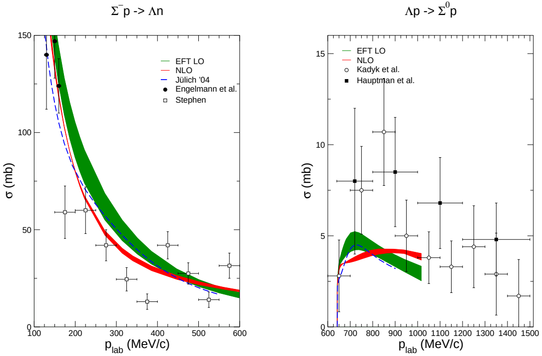
YN integrated cross sections



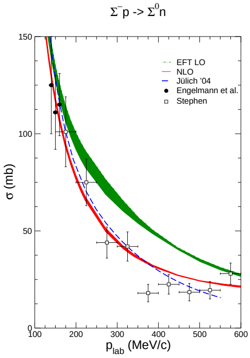
$\Sigma^+ p$ integrated cross section - new vs. old

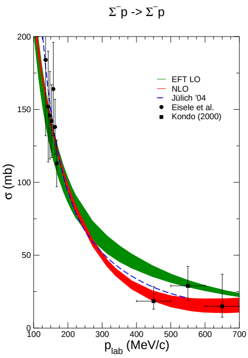


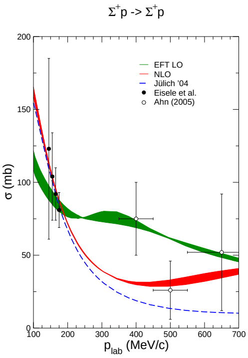
YN integrated cross sections - higher energies



YN integrated cross sections - higher energies







YN scattering lengths [fm]

	EFT LO	EFT NLO	Jülich '04	NSC97f	experiment
Λ [MeV]	$550 \cdots 700$	$500 \cdots 650$			
$a_s^{\Lambda p}$	$-1.90 \cdots -1.91$	$-2.90 \cdots -2.91$	-2.56	-2.51	$-1.8^{+2.3}_{-4.2}$
$a_t^{\Lambda p}$	$-1.22 \cdots -1.23$	$-1.51 \cdots -1.61$	-1.66	-1.75	$-1.6^{+1.1}_{-0.8}$
$a_s^{\Sigma^+ p}$	$-2.24 \cdots -2.36$	$-3.46 \cdots -3.60$	-4.71	-4.35	
$a_t^{\Sigma^+ p}$	$0.60 \cdots 0.70$	$0.48 \cdots 0.49$	0.29	-0.25	
χ^2	≈ 30	$15.7 \cdots 16.8$	≈ 25	16.7	
$({}^3_\Lambda\mathrm{H})\ E_B{}^\dagger$	$-2.34 \cdots -2.36$	$-2.30 \cdots -2.33$	-2.27	-2.30	-2.354(50)

Nuclear matter properties

conventional first-order Brueckner calculation:

Partial wave contributions to $-U_\Lambda(p_\Lambda = 0)$ (in MeV) at $k_F = 1.35\ \mathrm{fm}^{-1}$

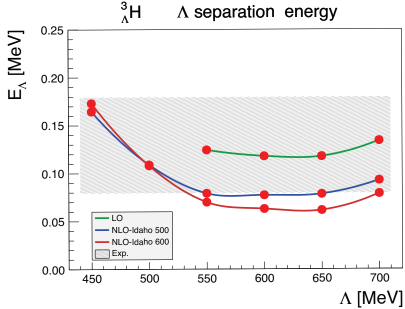
	$^1S_0$	$^3S_1 + ^3D_1$	$^3P_0$	$^1P_1 + ^3P_1$	$^3P_2 + ^3F_2$	Total
EFT LO	12.0	25.5	1.7	−3.3	0.4	36.5
EFT NLO	12.5	12.0	−0.9	−2.1	1.1	22.9
Jülich '04	9.9	35.0	0.7	0.2	3.3	49.7
Jülich '94	3.6	27.2	−0.6	−2.0	0.8	29.8
NSC97f	14.4	22.9	−0.5	−6.4	0.7	31.1

"Empirical" value for the Λ binding energy in nuclear matter:
$\approx$ 30 MeV

Nuclear matter properties

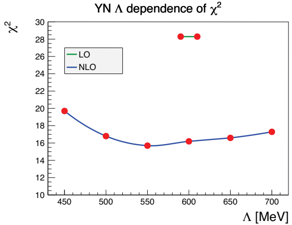
	EFT LO	EFT NLO	Jülich '04	Jülich '94	NSC97f
Λ [MeV]	$550 \cdots 700$	$500 \cdots 650$			
$-U_\Lambda(0)$	$38.0 \cdots 34.4$	$29.3 \cdots 22.9$	49.7	29.8	31.1
$-U_\Sigma(0)$	$-28.0 \cdots -11.1$	$-17.4 \cdots -12.1$	22.2	71.45	16.1

Hypertriton separation energies



separation energies:

$$E_\Lambda = E(core) - E(hypernucleus)$$

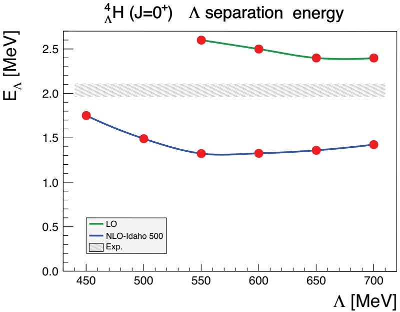


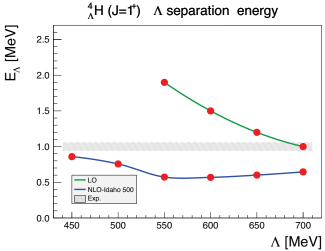
- singlet scattering length for one cutoff chosen so that hypertriton binding energy is OK

- cutoff variation
  - is **lower bound** for magnitude of higher order contributions
  - correlation with χ^2 of YN interaction ?

- long range 3BFs need to be explicitly estimated

Separation energies for $^{4}_{\Lambda}H$

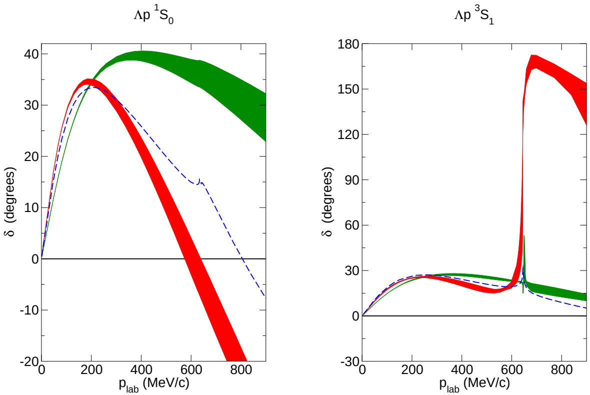




- LO/NLO results: LO uncertainty in 0^+ is underestimated by cutoff variation
- NLO results in line with model results, implies underbinding
- long range 3BFs need to be explicitly estimated

- **but:** for this version of NLO, results are **inconsistent** with experiment
  - note: this NLO does not allow for SU(3) breaking in contact part of YN
  - ad-hoc p-waves

Λp S-wave phase shifts



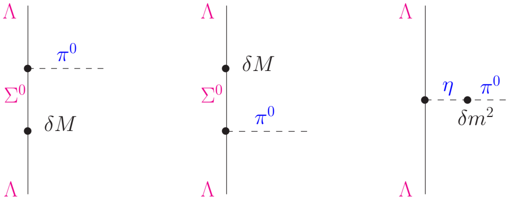
⇒ less repulsion in $^1S_0$ at short distances – and/or 3BFs ?

CSB at NLO & for model interactions

Contributions to the difference of ${}^4_\Lambda\mathrm{H}\,(0^+) - {}^4_\Lambda\mathrm{He}\,(0^+)$ separation energies

Λ [MeV]	450	500	550	600	650	700	Jülich 04	Nijm SC97	Nijm SC89	Expt.
ΔT [keV]	44	50	52	51	46	40	0	47	132	-
ΔV_{NN} [keV]	-3	-2	5	5	3	0	-31	-9	-9	-
ΔV_{YN} [keV]	-11	-11	-11	-10	-8	-7	2	37	228	-
tot [keV]	30	37	46	46	41	33	-29	75	351	350
$P_{\Sigma-}$	1.0%	1.1%	1.2%	1.2%	1.1%	0.9%	0.3%	1.0%	2.7%	-
$P_{\Sigma 0}$	0.6%	0.6%	0.7%	0.7%	0.6%	0.5%	0.3%	0.5%	1.4%	-
$P_{\Sigma+}$	0.1%	0.1%	0.2%	0.2%	0.2%	0.1%	0.3%	0.0%	0.1%	-

- **kinetic energy contribution is driven by Σ component**
- NN force contribution due to small deviation of Coulomb
- YN force contribution:
  - *SC89 CSB is strong*
  - *NLO CSB is zero, only Coulomb acts* (Σ component)



Electromagnetic mass matrix:

$\langle \Sigma^0 | \delta M | \Lambda \rangle = [M_{\Sigma^0} - M_{\Sigma^+} + M_p - M_n]/\sqrt{3}$

$\langle \pi^0 | \delta m^2 | \eta \rangle = [m^2_{\pi^0} - m^2_{\pi^+} + m^2_{K^+} - m^2_{K^0}]/\sqrt{3}$

(R.H. Dalitz & F. von Hippel, PL 10 (1964) 153)

$\Lambda - \Sigma^0$ mixing

$$f_{\Lambda\Lambda\pi} = [-2\frac{\langle\Sigma^0|\delta M|\Lambda\rangle}{M_{\Sigma^0}-M_\Lambda} + \frac{\langle\pi^0|\delta m^2|\eta\rangle}{m^2_\eta - m^2_{\pi^0}}]\, f_{\Lambda\Sigma\pi}$$

latest PDG mass values $\Rightarrow$

$$f_{\Lambda\Lambda\pi} \approx (-0.0297 - 0.0106)\, f_{\Lambda\Sigma\pi} \approx -0.0403\, f_{\Lambda\Sigma\pi}$$

Scattering lengths (in fm)

		Isospin basis	particle basis		+CSB	
		ΛN	Λp	Λn	Λp	Λn
EFT NLO (600)	$^1S_0$	-2.902	-2.906	-2.907	-2.866	-2.948
NSC97f		-1.60			-2.51	-2.68
EFT NLO (600)	$^3S_1$	-1.520	-1.541	-1.517	-1.547	-1.512
NSC97f		-1.72			-1.75	-1.66

Summary

YN interaction based on chiral *EFT*

- approach is based on a modified Weinberg power counting, analogous to the NN case
- The potential (contact terms, pseudoscalar-meson exchanges) is derived imposing $SU(3)_f$ constraints
- Good description of the empirical YN data was achieved already at LO (only 5 free parameters!)
- Excellent results at next-to-leading order (NLO)
- YN data are reproduced with a quality comparable to phenomenological models
- $SU(3)$ symmetry for the LEC's can be maintained in the YN system (ΛN, ΣN) but not between YN and NN

Outlook

Extension to N2LO

- same number of LECs that need to be determined
- employ spectral-function regularization like in the *NN* case
- in *NN*: cutoff $\tilde{\Lambda} \approx$ 500-700 MeV, i.e.

  $\tilde{\Lambda} \leq m_\pi + m_K,\ m_\pi + m_\eta$, ...

  $\Rightarrow$ should keep only $\pi\pi$ loops !

systematic investigation of *YNN* and *YNNN* systems

- binding energies are influenced by:

  (1) relative strength of the ΛN $^1S_0$ and $^3S_1$ interactions

  (2) strength of the ΛN–ΣN coupling ($^3S_1$–$^3D_1$)

  (3) possible three-body forces (beyond intermediate Σ)

  (appear formally at N2LO!)