# Effective Lagrangian for multi-baryon interactions

Stefan Petschauer
Technische Universität München

in collaboration with:

| | |
|---|---|
| Norbert Kaiser | Technische Universität München |
| Wolfram Weise | ECT* Trento, Technische Universität München |
| Johann Haidenbauer, Andreas Nogga | Forschungszentrum Jülich |
| Ulf-G. Meißner | Universität Bonn, Forschungszentrum Jülich |


TR110 $\mu$Workshop on Strangeness and related topics
Trento, December 5th, 2013

Work supported in part by DFG and NSFC (CRC110)

# Table of Contents

# Motivation

- Goal: determine YN and YY interactions
  - empirical constraints from YN scattering and Λ hypernuclei
  - strange baryons in nuclear matter

- accurate description of nuclear interactions with SU(2) B$\chi$PT

  [Epelbaum, Machleidt, . . .]

  extend SU(2) B$\chi$PT to include strangeness $\Rightarrow$ SU(3) B$\chi$PT

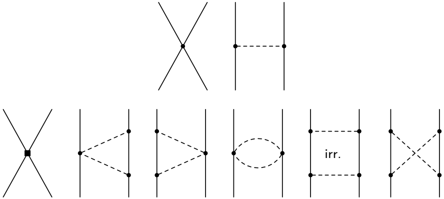
- Advantages:
  - improve results systematically
  - derive consistently two- and three-baryon forces

- Innovative work: YN and YY interactions in LO SU(3) B$\chi$PT by Jülich group

  [Polinder, Haidenbauer, Meißner, Nucl.Phys. A779, 2006]

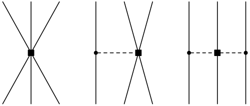
# Motivation

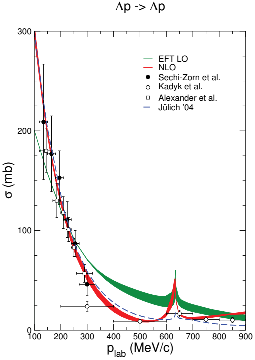
- systematic *NLO* analysis of *contact terms* and *one-* and *two-meson exchange* contributions to baryon-baryon interactions using SU(3) B$\chi$PT



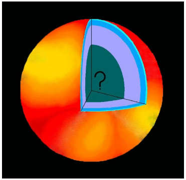

- repulsive *ΛNN* force suggested to get stiffer equation of state for neutron stars and to describe hypernuclei

[Bhaduri et al.,Ann.Phys.44,1967] [Gal et al.,Ann.Phys.63,1971] [Lonardoni et al.,Phys.Rev.C87,2013]





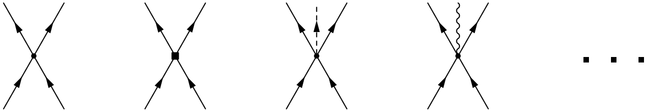

[Nucl.Phys.A915, 2013]



[ http://www.astro.umd.edu/~miller/nstar.html ]

# Table of Contents

# Construction of the Lagrangian

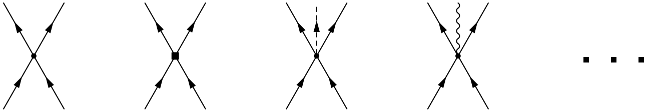


- external fields method for construction of Lagrangian [Gasser, Leutwyler]
- Lagrangian invariant under *local* transformations $\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}$
- $U(x) = \exp\left(\mathrm{i}\frac{\phi(x)}{f_0}\right) \equiv u^2(x)\,,\qquad \phi$ Goldstone boson octet

$$\phi=\begin{pmatrix} \pi^0+\frac{\eta}{\sqrt{3}} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0+\frac{\eta}{\sqrt{3}} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2\eta}{\sqrt{3}} \end{pmatrix}, \qquad B=\begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

$$U \to RUL^\dagger,\ u \to RuK^\dagger = KuL^\dagger, \quad B \to KBK^\dagger, \qquad K = K\,(L, R, U)$$

# Construction of the Lagrangian



- external fields method for construction of Lagrangian [Gasser, Leutwyler]
- Lagrangian invariant under *local* transformations $\mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}}$
- $U(x) = \exp\left(\mathrm{i}\frac{\phi(x)}{f_0}\right) \equiv u^2(x)\,,$ $\qquad \phi$ Goldstone boson octet

$$\phi = \begin{pmatrix} \pi^0 + \frac{\eta}{\sqrt{3}} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{\eta}{\sqrt{3}} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2\eta}{\sqrt{3}} \end{pmatrix}, \qquad B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

$$U \to R U L^\dagger,\ u \to R u K^\dagger = K u L^\dagger, \quad B \to K B K^\dagger, \qquad K = K(L, R, U)$$

- building blocks $u_\mu, \chi_+, \chi_-, f^+_{\mu\nu}, f^-_{\mu\nu}$ and baryon fields $B, \bar{B}$
  transform as $X \to K X K^\dagger$;
  same for covariant derivative $D_\mu X \to K(D_\mu X)K^\dagger$
- power counting [Krause, Helv.Phys.Acta 63, 1990]:
  $\mathcal{O}(p^0)$: $B,\ \bar{B},\ D_\mu B$; $\quad \mathcal{O}(p^1)$: $u_\mu,\ D_\mu$; $\quad \mathcal{O}(p^2)$: $f^+_{\mu\nu},\ f^-_{\mu\nu},\ \chi_+,\ \chi_-$

# Construction of the Lagrangian

- construct all terms in the Lagrangian by traces of products of building block, or products of such traces
- Sample structure: $\langle \bar{B}_1 \bar{B}_2 D\Gamma^1 B_1 D\Gamma^2 B_2 \dots \rangle$ $\langle \dots \rangle$: flavor trace

Used for simplification:

- invariance under Lorentz transformation, C, P, H, local $\mathrm{SU(3)_L \times SU(3)_R}$
- Fierz theorem
- equation of motion: $\mathrm{i}\not{D}B = M_0 B + \mathcal{O}(q)$
- cyclic property of trace
- SU(3) Cayley-Hamilton relation
- $[D_\mu, D_\nu]\, X = \frac{1}{4}\left[[u_\mu, u_\nu], X\right] - \frac{\mathrm{i}}{2}\left[f^+_{\mu\nu}, X\right]$

# Baryon-baryon contact terms up to NLO

**for pure baryon-baryon interactions:**

$$f^{\pm}_{\mu\nu} = 0, \qquad \chi_- = 0, \qquad \chi_+ = 4B_0\,\mathrm{diag}(m_u, m_d, m_s), \qquad D_\mu = \partial_\mu$$

- $\mathcal{O}(p^0)$: $\langle \bar{B}_1(\gamma_5\gamma_\mu B_1)\bar{B}_2(\gamma_5\gamma^\mu B_2)\rangle\,, \ldots$ (18 terms)
- $\mathcal{O}(p^1)$: (1 terms)

$$\hat{\partial}_2^\alpha \langle \bar{B}_1 \bar{B}_2 (\gamma_5\gamma_\alpha\partial_\mu B)_1(\gamma_5\gamma^\mu B)_2\rangle + \hat{\partial}_1^\alpha \langle \bar{B}_1(\partial_\mu \bar{B})_2(\gamma_5\gamma^\mu B)_1(\gamma_5\gamma_\alpha B)_2\rangle$$

$\Rightarrow$ antisymmetric spin-orbit term: $\mathrm{i}\,(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot(\vec{p}\times\vec{p}\,')$

$\Rightarrow$ spin singlet-triplet transitions: ${}^1P_1 \leftrightarrow {}^3P_1$

- $\mathcal{O}(p^2)$ (no external fields): $\langle \bar{B}_1 B_1 \partial^2(\bar{B}_2 B_2)\rangle\,, \ldots$ (9 terms)
- $\mathcal{O}(p^2)$ (with $\chi_+$): $\langle \bar{B}_1 \chi_+ B_1 \bar{B}_2 B_2\rangle\,, \ldots$ (12 terms)

# Contribution to partial waves

$$V(^1S_0) = \tilde{C}_{^1S_0} + C_{^1S_0}(p^2 + p'^2)$$
$$V(^3P_0) = C_{^3P_0}\, p\, p'$$
$$V(^3P_1) = C_{^3P_1}\, p\, p' \ ,$$
$$V(^3P_2) = C_{^3P_2}\, p\, p'$$

$$V(^3S_1) = \tilde{C}_{^3S_1} + C_{^3S_1}(p^2 + p'^2)$$
$$V(^1P_1) = C_{^1P_1}\, p\, p' \ ,$$
$$V(^3D_1 - {}^3S_1) = C_{^3S_1 - {}^3D_1}\, {p'}^2$$
$$V(^3S_1 - {}^3D_1) = C_{^3S_1 - {}^3D_1}\, p^2$$

$$\left.\begin{aligned} V(^3P_1 - {}^1P_1) &= C_{^3P_1 - {}^1P_1}\, p\, p' \\ V(^1P_1 - {}^3P_1) &= C_{^1P_1 - {}^3P_1}\, p\, p' \end{aligned}\right\} \quad \text{from antisym. spin-orbit term}$$

- as for NN interaction but with more constants because of different baryon-baryon channels
- $p$ ($p'$) incoming (outgoing) momentum in center-of-mass frame

| $S$ | $I$ | transition | $j \in \{{}^1S_0, {}^3P_0, {}^3P_1, {}^3P_2\}$ | $j \in \{{}^3S_1, {}^1P_1, {}^3S_1 \leftrightarrow {}^3D_1\}$ | ${}^1P_1 \to {}^3P_1$ | ${}^3P_1 \to {}^1P_1$ |
|---|---|---|---|---|---|---|
| $0$ | $0$ | $NN \to NN$ | $0$ | $c_j^{10^*}$ | $0$ | $0$ |
| | $1$ | $NN \to NN$ | $c_j^{27}$ | $0$ | $0$ | $0$ |
| $-1$ | $\frac{1}{2}$ | $\Lambda N \to \Lambda N$ | $\frac{1}{10}(9c_j^{27} + c_j^{8s})$ | $\frac{1}{2}(c_j^{10^*} + c_j^{8a})$ | $-c^{8as}$ | $-c^{8as}$ |
| | $\frac{1}{2}$ | $\Lambda N \to \Sigma N$ | $-\frac{3}{10}(c_j^{27} - c_j^{8s})$ | $\frac{1}{2}(c_j^{10^*} - c_j^{8a})$ | $-3c^{8as}$ | $c^{8as}$ |
| | $\frac{1}{2}$ | $\Sigma N \to \Sigma N$ | $\frac{1}{10}(c_j^{27} + 9c_j^{8s})$ | $\frac{1}{2}(c_j^{10^*} + c_j^{8a})$ | $3c^{8as}$ | $3c^{8as}$ |
| | $\frac{3}{2}$ | $\Sigma N \to \Sigma N$ | $c_j^{27}$ | $c_j^{10}$ | $0$ | $0$ |
| $-2$ | $0$ | $\Lambda\Lambda \to \Lambda\Lambda$ | $\frac{1}{40}(5c_j^{1} + 27c_j^{27} + 8c_j^{8s})$ | $0$ | $0$ | $0$ |
| | $0$ | $\Lambda\Lambda \to \Xi N$ | $\frac{1}{20}(5c_j^{1} - 9c_j^{27} + 4c_j^{8s})$ | $0$ | $0$ | $2c^{8as}$ |
| | $0$ | $\Lambda\Lambda \to \Sigma\Sigma$ | $-\frac{\sqrt{3}}{40}(5c_j^{1} + 3c_j^{27} - 8c_j^{8s})$ | $0$ | $0$ | $0$ |
| | $0$ | $\Xi N \to \Xi N$ | $\frac{1}{10}(5c_j^{1} + 3c_j^{27} + 2c_j^{8s})$ | $c_j^{8a}$ | $2c^{8as}$ | $2c^{8as}$ |
| | $0$ | $\Xi N \to \Sigma\Sigma$ | $\frac{\sqrt{3}}{20}(-5c_j^{1} + c_j^{27} + 4c_j^{8s})$ | $0$ | $2\sqrt{3}c^{8as}$ | $0$ |
| | $0$ | $\Sigma\Sigma \to \Sigma\Sigma$ | $\frac{1}{40}(15c_j^{1} + c_j^{27} + 24c_j^{8s})$ | $0$ | $0$ | $0$ |
| | $1$ | $\Xi N \to \Xi N$ | $\frac{1}{5}(2c_j^{27} + 3c_j^{8s})$ | $\frac{1}{3}(c_j^{10} + c_j^{10^*} + c_j^{8a})$ | $-2c^{8as}$ | |
| | $1$ | $\Xi N \to \Sigma\Sigma$ | $0$ | $\frac{1}{3\sqrt{2}}(c_j^{10} + c_j^{10^*} - 2c_j^{8a})$ | $0$ | $2\sqrt{2}c^{8as}$ |
| | $1$ | $\Xi N \to \Sigma\Lambda$ | $\frac{\sqrt{6}}{5}(c_j^{27} - c_j^{8s})$ | $\frac{1}{\sqrt{6}}(c_j^{10} - c_j^{10^*})$ | $2\sqrt{\frac{2}{3}}c^{8as}$ | $0$ |
| | $1$ | $\Sigma\Lambda \to \Sigma\Lambda$ | $\frac{1}{5}(3c_j^{27} + 2c_j^{8s})$ | $\frac{1}{2}(c_j^{10} + c_j^{10^*})$ | $0$ | |
| | $1$ | $\Sigma\Lambda \to \Sigma\Sigma$ | $0$ | $\frac{1}{2\sqrt{3}}(c_j^{10} - c_j^{10^*})$ | $0$ | $\frac{4}{\sqrt{3}}c^{8as}$ |
| | $1$ | $\Sigma\Sigma \to \Sigma\Sigma$ | $0$ | $\frac{1}{6}(c_j^{10} + c_j^{10^*} + 4c_j^{8a})$ | $0$ | $0$ |
| | $2$ | $\Sigma\Sigma \to \Sigma\Sigma$ | $c_j^{27}$ | $0$ | $0$ | $0$ |
| $-3$ | $\frac{1}{2}$ | $\Xi\Lambda \to \Xi\Lambda$ | $\frac{1}{10}(9c_j^{27} + c_j^{8s})$ | $\frac{1}{2}(c_j^{10} + c_j^{8a})$ | $-c^{8as}$ | $-c^{8as}$ |
| | $\frac{1}{2}$ | $\Xi\Lambda \to \Xi\Sigma$ | $-\frac{3}{10}(c_j^{27} - c_j^{8s})$ | $\frac{1}{2}(c_j^{10} - c_j^{8a})$ | $-3c^{8as}$ | $c^{8as}$ |
| | $\frac{1}{2}$ | $\Xi\Sigma \to \Xi\Sigma$ | $\frac{1}{10}(c_j^{27} + 9c_j^{8s})$ | $\frac{1}{2}(c_j^{10} + c_j^{8a})$ | $3c^{8as}$ | $3c^{8as}$ |
| | $\frac{3}{2}$ | $\Xi\Sigma \to \Xi\Sigma$ | $c_j^{27}$ | $c_j^{10^*}$ | $0$ | $0$ |
| $-4$ | $0$ | $\Xi\Xi \to \Xi\Xi$ | $0$ | $c_j^{10}$ | $0$ | $0$ |
| | $1$ | $\Xi\Xi \to \Xi\Xi$ | $c_j^{27}$ | $0$ | $0$ | $0$ |

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{8_s} \oplus \mathbf{1} \oplus \mathbf{10} \oplus \mathbf{10^*} \oplus \mathbf{8_a}$$

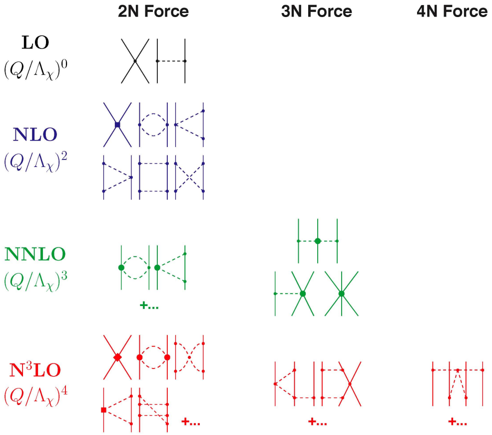
[Polinder, Haidenbauer, Meißner, Nucl.Phys.A779, 2006]
[Petschauer, Kaiser, Nucl.Phys.A916, 2013]

| $S$ | $I$ | transition | $^1S_0\ \chi$ | $^3S_1\ \chi$ |
|---|---|---|---|---|
| 0 | 0 | $NN \to NN$ | $0$ | $\frac{c_\chi^7}{2}$ |
| | 1 | $NN \to NN$ | $\frac{c_\chi^1}{2}$ | $0$ |
| −1 | $\frac{1}{2}$ | $\Lambda N \to \Lambda N$ | $c_\chi^2$ | $c_\chi^8$ |
| | $\frac{1}{2}$ | $\Lambda N \to \Sigma N$ | $-c_\chi^3$ | $-c_\chi^9$ |
| | $\frac{1}{2}$ | $\Sigma N \to \Sigma N$ | $c_\chi^4$ | $c_\chi^{10}$ |
| | $\frac{3}{2}$ | $\Sigma N \to \Sigma N$ | $\frac{c_\chi^1}{4}$ | $-\frac{c_\chi^7}{4}$ |
| −2 | 0 | $\Lambda\Lambda \to \Lambda\Lambda$ | $\frac{c_\chi^5}{2}$ | $0$ |
| | 0 | $\Lambda\Lambda \to \Xi N$ | $\frac{3c_\chi^1}{4} - 3c_\chi^2 - c_\chi^3 + \frac{3c_\chi^5}{4}$ | $0$ |
| | 0 | $\Lambda\Lambda \to \Sigma\Sigma$ | $0$ | $0$ |
| | 0 | $\Xi N \to \Xi N$ | $\frac{2c_\chi^1}{3} - 3c_\chi^2 + \frac{c_\chi^4}{3} + \frac{9c_\chi^5}{8}$ | $c_\chi^{11}$ |
| | 0 | $\Xi N \to \Sigma\Sigma$ | $-\frac{c_\chi^1}{4\sqrt{3}} + \sqrt{3}c_\chi^3 + \frac{c_\chi^4}{\sqrt{3}}$ | $0$ |
| | 0 | $\Sigma\Sigma \to \Sigma\Sigma$ | $0$ | $0$ |
| | 1 | $\Xi N \to \Xi N$ | $c_\chi^6$ | $c_\chi^{12}$ |
| | 1 | $\Xi N \to \Sigma\Sigma$ | $0$ | $\sqrt{2}c_\chi^{10} - \frac{c_\chi^7}{2\sqrt{2}} - \sqrt{2}c_\chi^9$ |
| | 1 | $\Xi N \to \Sigma\Lambda$ | $-\frac{1}{3}\sqrt{\frac{2}{3}}c_\chi^1 + \sqrt{\frac{3}{2}}c_\chi^2 - \frac{c_\chi^4}{3\sqrt{6}} - \sqrt{\frac{2}{3}}c_\chi^6$ | $\frac{c_\chi^{10}}{\sqrt{6}} + \sqrt{\frac{2}{3}}c_\chi^{12} + \frac{c_\chi^7}{2\sqrt{6}} - \sqrt{\frac{3}{2}}c_\chi^8 + \sqrt{\frac{2}{3}}c_\chi^9$ |
| | 1 | $\Sigma\Lambda \to \Sigma\Lambda$ | $-\frac{c_\chi^1}{9} + \frac{4c_\chi^3}{3} + \frac{4c_\chi^4}{9} + \frac{2c_\chi^6}{3}$ | $\frac{4c_\chi^{10}}{3} + \frac{2c_\chi^{12}}{3} - \frac{c_\chi^7}{3} - \frac{4c_\chi^9}{3}$ |
| | 1 | $\Sigma\Lambda \to \Sigma\Sigma$ | $0$ | $0$ |
| | 1 | $\Sigma\Sigma \to \Sigma\Sigma$ | $0$ | $0$ |
| | 2 | $\Sigma\Sigma \to \Sigma\Sigma$ | $0$ | $0$ |
| −3 | $\frac{1}{2}$ | $\Xi\Lambda \to \Xi\Lambda$ | $-\frac{55c_\chi^1}{72} + 2c_\chi^2 + \frac{7c_\chi^3}{6} + \frac{c_\chi^4}{18} + \frac{3c_\chi^5}{32} + \frac{c_\chi^6}{12}$ | $\frac{11c_\chi^{10}}{12} + \frac{3c_\chi^{11}}{4} + \frac{25c_\chi^{12}}{12} + \frac{5c_\chi^7}{24} - \frac{7c_\chi^8}{4} - \frac{c_\chi^9}{6}$ |
| | $\frac{1}{2}$ | $\Xi\Lambda \to \Xi\Sigma$ | $\frac{11c_\chi^1}{24} - \frac{3c_\chi^2}{2} - \frac{c_\chi^3}{2} - \frac{c_\chi^4}{3} + \frac{9c_\chi^5}{32} + \frac{c_\chi^6}{4}$ | $\frac{9c_\chi^{10}}{4} - \frac{3c_\chi^{11}}{4} + \frac{5c_\chi^{12}}{4} - \frac{c_\chi^7}{8} - \frac{3c_\chi^8}{4} - \frac{c_\chi^9}{2}$ |
| | $\frac{1}{2}$ | $\Xi\Sigma \to \Xi\Sigma$ | $\frac{11c_\chi^1}{24} - 3c_\chi^2 + \frac{5c_\chi^3}{2} + \frac{c_\chi^4}{6} + \frac{27c_\chi^5}{32} + \frac{3c_\chi^6}{4}$ | $\frac{5c_\chi^{10}}{4} + \frac{3c_\chi^{11}}{4} + \frac{3c_\chi^{12}}{4} - \frac{c_\chi^7}{8} - \frac{3c_\chi^8}{4} - \frac{3c_\chi^9}{2}$ |
| | $\frac{3}{2}$ | $\Xi\Sigma \to \Xi\Sigma$ | $-\frac{2c_\chi^1}{3} + \frac{3c_\chi^2}{2} + c_\chi^3 + \frac{c_\chi^4}{6}$ | $\frac{3c_\chi^{10}}{2} - c_\chi^7 + \frac{3c_\chi^8}{2} - 3c_\chi^9$ |
| −4 | 0 | $\Xi\Xi \to \Xi\Xi$ | $0$ | $5c_\chi^{10} + 4c_\chi^{12} - 3c_\chi^8 - 2c_\chi^9$ |
| | 1 | $\Xi\Xi \to \Xi\Xi$ | $-\frac{4c_\chi^1}{3} + 3c_\chi^2 + 2c_\chi^3 + \frac{c_\chi^4}{3}$ | $0$ |

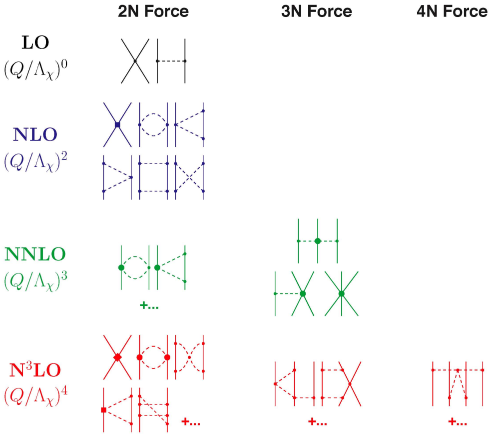
[Petschauer, Kaiser, Nucl.Phys.A916, 2013]

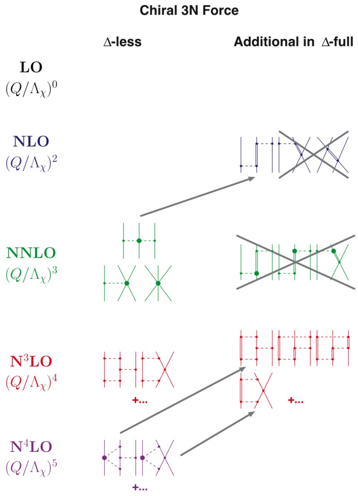
# Table of Contents

# Nuclear forces



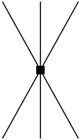
# Nuclear forces



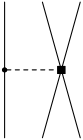




[Machleidt, Entem, Phys.Rept.503 (2011)]

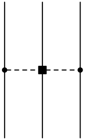
# Leading order three-nucleon forces



$$V^{3NF}_{cont} = \frac{1}{2} E \sum_{j \neq k} \vec{\tau}_j \cdot \vec{\tau}_k$$



$$V^{3NF}_{OPE} = -\frac{g_A}{8 f_\pi^2} D \sum_{i \neq j \neq k} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{\vec{q}_j^{\,2} + m_\pi^2} \vec{\tau}_i \cdot \vec{\tau}_j \; \vec{\sigma}_i \cdot \vec{q}_j$$



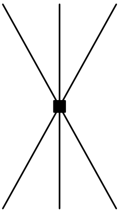
$$V^{3NF}_{TPE} = \frac{g_A^2}{8 f_\pi^2} \sum_{i \neq j \neq k} \frac{\vec{\sigma}_i \cdot \vec{q}_i \vec{\sigma}_j \cdot \vec{q}_j}{(q_i^2 + m_\pi^2)(q_j^2 + m_\pi^2)} F^{\alpha\beta}_{ijk} \tau_i^\alpha \tau_j^\beta$$

$$F^{\alpha\beta}_{ijk} = \frac{\delta^{\alpha\beta}}{f_\pi^2} (-4 c_1 m_\pi^2 + 2 c_3 \vec{q}_i \cdot \vec{q}_j) + \sum_\gamma \frac{c_4}{f_\pi^2} \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \; \vec{\sigma}_k \cdot (\vec{q}_i \times \vec{q}_j)$$

$p(p')$ are initial (final) momenta of the nucleon i and $\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i$

[Epelbaum, Nogga, Glöckle, Kamada, Meißner, Witała, Phys.Rev.C66, 2002]

# Short-range three-baryon force



- $B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$

- possible Dirac structures
  $\mathbb{1}, \gamma_\mu, \gamma_5, \gamma_5\gamma_\mu, \sigma_{\mu\nu}$

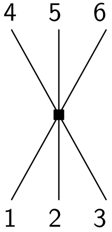
- leads after non-relativistic expansion to potentials of the form
  $\mathbb{1}\,,\ \vec{\sigma}_1 \cdot \vec{\sigma}_2\,,\ \vec{\sigma}_1 \cdot \vec{\sigma}_3\,,\ \vec{\sigma}_2 \cdot \vec{\sigma}_3\,,\ \vec{\sigma}_1 \times \vec{\sigma}_2 \cdot \vec{\sigma}_3$

- Lagrangian terms

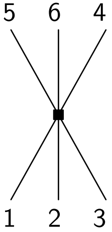
$$\langle \bar{B}\bar{B}\bar{B}BBB \rangle$$
$$\langle \bar{B}\bar{B}B\bar{B}BB \rangle$$
$$\langle \bar{B}\bar{B}BB\bar{B}B \rangle$$
$$\langle \bar{B}B\bar{B}B\bar{B}B \rangle$$
$$\langle \bar{B}\bar{B}\bar{B}B \rangle\langle BB \rangle \pm \langle \bar{B}\bar{B} \rangle\langle \bar{B}BBB \rangle$$
$$\langle \bar{B}\bar{B}BB \rangle\langle \bar{B}B \rangle$$
$$\langle \bar{B}B\bar{B}B \rangle\langle \bar{B}B \rangle$$
$$\langle \bar{B}\bar{B}\bar{B} \rangle\langle BBB \rangle$$
$$\langle \bar{B}\bar{B}B \rangle\langle \bar{B}BB \rangle$$
$$\langle \bar{B}\bar{B} \rangle\langle \bar{B}B \rangle\langle BB \rangle$$
$$\langle \bar{B}B \rangle\langle \bar{B}B \rangle\langle \bar{B}B \rangle$$

$\langle \ldots \rangle$: flavor trace

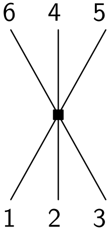
$V^{B_1 B_2 B_3 \to B_4 B_5 B_6} =$



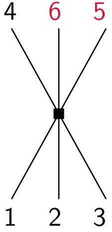

$+ P_{13} P_{23}$



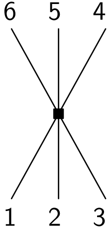

$+ P_{12} P_{23}$



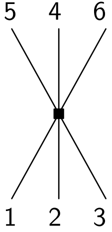

$- P_{23}$




$- P_{13}$




$- P_{12}$




with spin exchange operator $P_{ij} = \frac{1}{2}(1 + \vec{\sigma}_i \cdot \vec{\sigma}_j)$

# Preliminary results for three baryon contact terms

contact terms in different strangeness sectors:

| strangeness | parameters |
|---|---|
| 0 | 1 parameter |
| -1 | additional 7 parameters |
| -2 | additional 9 parameters |
| -3 | additional 1 parameters |
| -4 | no additional parameters |
| -5 | no additional parameters |
| -6 | no additional parameters |

$\Rightarrow$ in total 18 parameters

## $\Lambda NN$ interaction

Isospin $I = 0$: $V^{I=0}_{\Lambda NN \to \Lambda NN} = e_2(\mathbb{1} + \frac{1}{3}\,\vec{\sigma}_2 \cdot \vec{\sigma}_3) + e_3(\vec{\sigma}_1 \cdot \vec{\sigma}_2 + \vec{\sigma}_1 \cdot \vec{\sigma}_3)$

Isospin $I = 1$: $V^{I=1}_{\Lambda NN \to \Lambda NN} = e_4(\mathbb{1} - \,\vec{\sigma}_2 \cdot \vec{\sigma}_3)$

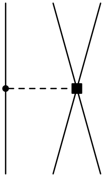
# Checks

- conservation of strangeness $S$ and isospin $I$
- conservation and independence of isospin $I_3$
- time reversal symmetry
- reproduce NNN contact term $V_{\rm ct}^{\rm 3NF} = E \frac{1}{2} \sum_{i \neq j} \vec{\tau}_i \cdot \vec{\tau}_j$
- reordering particles in final and initial state
  (spin-exchange operators and Racah recoupling)
- group theory:
  totally antisymmetric part of $\mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8}$ in flavor space:

$$\text{Antisym}_3(\mathbf{8}) = \mathbf{56} = \mathbf{27} + \mathbf{10} + \mathbf{10}^\star + \mathbf{8} + \mathbf{1}$$

$\Rightarrow$ consistent with 5 LEC's for spin 3/2

# Mid-range three-baryon force



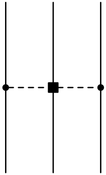
$$u_\mu = -\frac{1}{f_0}\partial_\mu \phi$$

$$\begin{aligned}\mathcal{L}_{BB}^{(2)} = \quad & d_1\Big(\langle \bar{B}Bu_\mu \bar{B}\gamma_5\gamma^\mu B\rangle + \langle \bar{B}B\bar{B}\gamma_5\gamma^\mu B u_\mu\rangle\Big) \\ & + \mathrm{i} d_2\Big(\langle \bar{B}\gamma_5\gamma_\nu B u_\mu \bar{B}\sigma^{\mu\nu}B\rangle - \langle \bar{B}\gamma_5\gamma_\nu B\bar{B}\sigma^{\mu\nu}Bu_\mu\rangle\Big) \\ & + \ldots\end{aligned}$$

[Petschauer, Kaiser, Nucl.Phys.A916, 2013]

$$V \propto \frac{1}{f_0}\frac{\vec{\sigma}_1\cdot\vec{q}_1}{\vec{q}_1^{\,2}+m_\phi^2}\Big\{\ \vec{\sigma}_2\cdot\vec{q}_1\ ,\ \vec{\sigma}_3\cdot\vec{q}_1\ ,\ (\vec{\sigma}_2\times\vec{\sigma}_3)\cdot\vec{q}_1\ \Big\}$$

# Long-range three-baryon force



$$u_\mu = -\tfrac{1}{f_0}\partial_\mu \phi\,, \quad \chi_+ = 2\chi - \tfrac{1}{f_0^2}\{\phi,\{\phi,\chi\}\}$$

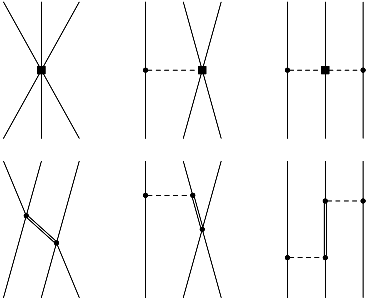
$$\begin{aligned}\mathcal{L}^{(2)}_{MB} = \quad & b_D\langle \bar{B}\{\chi_+, B\}\rangle + b_F\langle \bar{B}[\chi_+, B]\rangle + b_0\langle \bar{B}B\rangle\,\langle\chi_+\rangle \\ & + b_1\langle \bar{B}[u^\mu,[u_\mu,B]]\rangle + b_2\langle \bar{B}\{u^\mu,\{u_\mu,B\}\}\rangle \\ & + b_3\langle \bar{B}\{u^\mu,[u_\mu,B]\}\rangle + b_4\langle \bar{B}B\rangle\,\langle u^\mu u_\mu\rangle \\ & + \mathrm{i}d_1\langle \bar{B}\{[u^\mu,u^\nu],\sigma_{\mu\nu}B\}\rangle + \mathrm{i}d_2\langle \bar{B}[[u^\mu,u^\nu],\sigma_{\mu\nu}B]\rangle \\ & + \mathrm{i}d_3\langle \bar{B}u^\mu\rangle\langle u^\nu\sigma_{\mu\nu}B\rangle\end{aligned}$$

[Oller, Verbeni, Prades, JHEP 0609, 2006]

$$V \propto \frac{1}{f_0^2}\frac{\vec{\sigma}_1\cdot\vec{q}_1\;\vec{\sigma}_3\cdot\vec{q}_3}{(\vec{q}_1^{\,2}+m^2_{\phi_1})(\vec{q}_3^{\,2}+m^2_{\phi_3})}\left\{\;\vec{q}_1\cdot\vec{q}_3\;,\;\; m_\pi^2\;,\;\; m_K^2\;,\;\; \vec{\sigma}_2\cdot(\vec{q}_1\times\vec{q}_3)\;\right\}$$

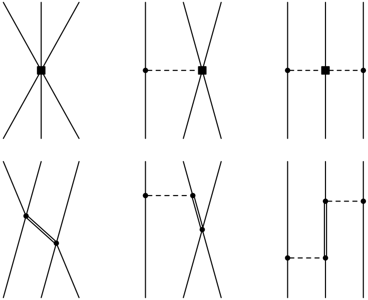
# Include decuplet baryons

- estimate LEC's by resonance saturation with decuplet baryons

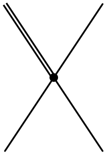


# Include decuplet baryons

- estimate LEC's by resonance saturation with decuplet baryons



- special vertex



$\mathbf{10} \otimes \mathbf{8} = \mathbf{35} \oplus \mathbf{27} \oplus \mathbf{10} \oplus \mathbf{8}$ $\qquad$ $\mathbf{3/2} \otimes \mathbf{1/2} = \mathbf{1} \oplus \mathbf{2}$

Tensor products in flavor space $\qquad$ and spin space

$\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{8_s} \oplus \mathbf{1} \oplus \mathbf{10} \oplus \mathbf{10}^* \oplus \mathbf{8_a}$ $\qquad$ $\mathbf{1/2} \otimes \mathbf{1/2} = \mathbf{0} \oplus \mathbf{1}$

# Table of Contents

# Summary

- SU(3) chiral effective field theory for hyperon-nucleon potentials
- NLO analysis of one- and two-meson exchange and contact terms with SU(3) symmetric LECs [Nucl.Phys. A915, 2013]
- good description of available YN data; comparable to phenomenological models
- complete classification of NLO baryon-baryon contact Lagrangian including external fields available [Nucl.Phys. A916, 2013]
- SU(3) classification of leading order three-baryon contact terms

# Outlook

- include two-meson exchange with intermediate *decuplet* baryons
- include *explicit* SU(3) symmetry breaking in contact terms
- future applications: hypernuclei, exotic neutron star matter,
  hyperons in nuclear matter ($\Sigma, \Lambda$ mean-fields)
- estimate strength of three-baryon forces in SU(3) B$\chi$PT
  by resonance saturation